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Quantum computing : a gentle introduction for AI engineers.

1. Quantum computing. 2. Computer algorithms. 3. Machine learning.
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